

Computability and Logic

HW 5

Due: Friday, March 20

1. Use Fitch to provide formal proofs for the following set theoretic theorems. Send your proofs to the Grade Grinder.
 - a. Problem 15.13: For any set A : $\emptyset \subseteq A$
 - b. Problem 15.15: For any sets A and B : $A = B$ iff $A \subseteq B$ and $B \subseteq A$
 - c. Problem 15.16: For any sets A , B , and C : if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
 - d. Problem 15.23: For any sets A and B : $A \cup B = B$ iff $A \subseteq B$
2. Prove that any endo-relation R is asymmetric if and only if R is anti-symmetric and irreflexive in the following 3 ways:
 - a. By providing a mathematical proof
 - b. By providing a Fitch proof for Fitch file 2b.prf as provided on LMS
 - c. By providing a Fitch proof for Fitch file 2c.prf as provided on LMS
3. Prove that if R is a binary relation, and R^{-1} its inverse (see LPL book section 15.5 for mathematical definition of inverse): R is Left-Total if and only if R^{-1} is Right-Total. Do this by providing the Fitch proofs for 3a.prf, 3b.prf, 3c.prf, and 3d.prf as provided on LMS. Use 3a as a Lemma in 3c, and use both 3b and 3c as Lemma's in 3d.
4. Define for any sets A and B : $|A| \leq |B|$ if and only if there exists a total and one-to-one function from A to B . Prove (mathematically) that for any sets A , B , and C :
If $|A| \leq |B|$ and $|B| \leq |C|$, then $|A| \leq |C|$
5. Provide mathematical proofs (from scratch!) for the following theorems:
 - a. the set of all finite subsets of the set of natural numbers is enumerable.
 - b. the set of all subsets of the set of natural numbers is not enumerable.